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1994 J. Phys.: Condens. Matter 6 10783

(http://iopscience.iop.org/0953-8984/6/49/019)

J. Phys.: Condens. Matter 6 (1994) 10783–10793. Printed in the UK

Flux-flow conductivity in high-T_c superconductors

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Received 24 August 1994

Abstract. Using an early theory by Houghton and Maki based on the BCS model, which assumes a temperature-independent relaxation time τ, we compute the flux-flow magnetoresistivity $\rho_s(T, H)$ of layered superconductors near H_{c_2} in the clean limit. Then we use our results to analyse the magnetoresistivity curves of single-crystal YBaCuO. Extending the validity of the calculation to a temperature-dependent τ, we find that for constant τ, or for relaxation times obtained from experiments, the theory reproduces qualitatively the saturation-like behaviour of the observed magnetoresistance, and also the broadening of the resistive transition, as well as the changes in the slope $\mathrm{d}\rho_s(T)/\mathrm{d}T$ with field. However, quantitative agreement with experiments is not achieved. This allows an estimate to be made of the contribution of the thermal vortex line fluctuations to the total dissipation; we find the former to be about 10% of the latter.

1. Introduction

The microscopic theory of flux-flow conductivity in pure type-II superconductors was analysed in early work by Maki and Houghton [1, 2]. Using the BCS theory and the Green-function formalism, these authors derived the expression for the flux-flow conductivity in high field at 0 K for conventional 3D superconductors. Their results have led to a better agreement between theory and experiments. To our knowledge, no extension of this work to other cases in the framework of BCS theory has since been carried out.

The mixed phase of high-temperature superconductors is characterized by a broad resistive transition. Flux-flow conductivity in that context has been discussed essentially from a phenomenological point of view, based on the Ginzburg–Landau-functional approach [3]. This approach seems reasonable in view of the complexity of a microscopic description of the vortex liquid (VL) state. A VL is most likely the correct description of the mixed phase in cuprates at temperatures sufficiently close to the superconducting temperature. The phenomenological approach, however, leaves aside a number of questions. For one thing, it is appropriate in the dirty limit, when the mean free path l is small compared to the Pippard coherence length $\xi_a(0)$, while superconducting cuprates are in the clean limit, $l \gg \xi_a(0)$ [4–6]. For another, the viscosity η, which is basic to the description of transport phenomena in the mixed phase in the absence of pinning, is assumed to be related to the normal-state resistivity ρ_n. This assumption is valid in conventional superconductors, where the low-temperature normal-resistivity state is dominated by elastic scattering processes due to impurities and is temperature independent. In cuprates, however, the normal-state resistivity is strongly temperature dependent, indicating that inelastic scattering processes are at work, most probably due to a spin-fluctuation phenomenon [7], related to the very microscopic phenomenon of superconductivity in cuprates [8]. The temperature dependence

0953-8984/94/4910783+11$19.50

of the vortex viscosity is in fact an open question at this time; its solution depends on a reliable microscopic theory of high-temperature superconductivity, about which the battle is still raging. It is very likely that in cuprates the appearance of superconductivity leads to important changes in the quasiparticle scattering rate and its temperature dependence below T_c. Experiments on YBCO exhibit a large enhancement of the real conductivity $\sigma_1(\omega)$ at low frequency [9], which points to a dramatic increase of quasiparticle lifetime below T_c. This question is in turn related to the symmetry of the superconducting order parameter; Ginzburg–Landau expansions, usually, as well as BCS theory, assume s symmetry, while experiments point to a d wave [10].

A fully reliable microscopic theory of flux-flow conductivity in cuprates depends on a reliable microscopic theory of the superconducting mechanism itself, which would presumably encompass the microscopic theory of transport phenomena in the normal phase, as well as a description of the quasiparticles in the condensed phase [8].

This paper is an attempt (far from completely satisfactory) at bridging the gap between phenomenological and microscopic theories, by extending Houghton and Maki's work to pure layered systems at finite temperatures in the clean limit, assuming an s-wave order parameter. This admittedly does not answer all the demands of a reliable theory; the model is based on a temperature-independent (elastic) impurity scattering process and on a Green-function approach, which describes an average regular vortex line lattice (VLL); the dynamic fluctuations of the order parameter we take into account do not correspond to a VL, but to the dynamic fluctuations of the order parameter in an ordered VLL.

Our hope is that it is nevertheless meaningful to tinker somewhat and extend the validity of such a treatment to a temperature-dependent $\tau(T)$, due to inelastic scattering processes which we do not take into account in a consistent way. What we find is that for any τ we choose, the theory qualitatively accounts for the broadening of the resistive transition and the changes in the slope $\mathrm{d}\rho_s(T)/\mathrm{d}T$ with field as well as the saturation-like behaviour of the observed magnetoresistance near H_{c_2}. However our approach is unable to account quantitatively for the observed dissipation. The 'most' dissipative' result (i.e. that which corresponds to the shortest τ) yields a conductivity gain twice larger than the observed one. We are naturally tempted to ascribe the unaccounted dissipation to the vortex line fluctuations. Thus this latter term is estimated to be roughly from 5% to 15% of the total dissipation.

The remainder of the paper is divided into four sections: in section 2, we derive the theoretical expression of the flux-flow conductivity of layered compounds in magnetic fields H slightly smaller than $H_{c_2}(T)$ using the theory of Houghton and Maki. Then, in section 3, we evaluate numerically the theoretical expression of the flux-flow conductivity. In section 4, we use our model to analyse the magnetoresistivity curves of single-crystal YBCO. Section 5 is devoted to some concluding remarks.

2. Flux-flow conductivity in layered compounds

We consider a layered superconductor made up of superconducting planes packed along an axis we shall refer to as the c axis. We also assume a configuration where the magnetic field is applied along the c axis, and the electric field is applied in the a–b plane. Taking into account the dynamical fluctuation of the order parameter, the flux-flow conductivity can be written as a sum of two contributions [2] (see the appendix):

$$\sigma_s = \sigma^{st} + \sigma^{fl} \tag{1}$$

where the static part σ^{st} and the fluctuation part σ^{fl} of σ can be expressed in terms of the correlation functions $\langle[j_x, j_x]\rangle$, $\langle[j_x, \Psi]\rangle$ and $\langle[\Psi, \Psi^+]\rangle$ as follows:

$$\sigma^{st} = A_{jj} \tag{2}$$

$$\sigma^{fl} = 4A_{j\Psi}\{D_1(0)\langle[\Psi, j_x]\rangle(0)\} \tag{3}$$

where A_{jj}, $A_{j\Psi}$ and $D_1(0)$ are given by

$$\langle[j, j]\rangle(-i\omega) = \langle[j, j]\rangle(0) - i\omega A_{jj} + O(\omega^2) \tag{4}$$

$$\langle[j, \Psi]\rangle(-i\omega) = \langle[j, \Psi]\rangle(0) - i\omega A_{j\Psi} + O(\omega^2) \tag{5}$$

$$D_1(0) = |g|/[1 - |g|\langle[\Psi, \Psi^+]\rangle(0)]. \tag{6}$$

g is the BCS coupling constant. The correlator of equation (5) expresses the coupling between the current and the fluctuations of the order parameter. We refer the reader to the appendix and to [11] and [12] for the details leading to the derivation of the above set of equations. In order to evaluate A_{jj}, we consider the isotropic 3D expression for A_{jj} given in [2] and apply it to our case. Thus we take $\sin\theta = 1$, where θ is the angle between the momentum $\boldsymbol{p}$ and the magnetic field $\boldsymbol{H}$, and we have

$$A_{jj} = \frac{e^2 p_F^3}{(2\pi)^3 m^2 v} \operatorname{Re}\left\{\int_0^\infty \frac{d\omega}{2T} \operatorname{sech}^2\left(\frac{\omega}{2T}\right)[I_1 + I_2]\right\} \tag{7}$$

where

$$I_1 = \int d\xi\, G(\xi, \omega - i\delta)G(\xi, \omega + i\delta) \tag{8}$$

and

$$I_2 = \Delta^2 \int d\xi\, du\, \rho(u)F(\xi, u, \omega + i\delta)F(\xi, u, \omega - i\delta) \tag{9}$$

with $\rho(u) = (\varepsilon\sqrt{\pi})^{-1}e^{-(u/\varepsilon)^2}$ and $\varepsilon = kv_F$. $k = \sqrt{2eH/\hbar}$ is the reciprocal lattice vector of the flux-line lattice; $\xi = v_a(p - p_F)$; d is the interlayer distance.

Following Houghton and Maki, we are making use of the Brandt, Pesch and Tewordt (BPT) Green functions in our expressions [13]. Brandt, Pesch and Tewordt have derived an approximate expression of the Green function for pure type-II superconductors near H_{c_2} in the case where the space variations of the order parameter are neglected by disregarding the small components for $k \neq 0$ with respect to that at $k = 0$ in the Fourier-series development of the order parameter. This approximation becomes very good in the London limit, i.e. when $\kappa \gg 1$ where $\kappa = \lambda(0)/\xi(0)$. $\lambda(0)$ and $\xi(0)$ are respectively the penetration length and the coherence length. We will suppose that this limit is relevant for our system. The BPT Green function G is given by

$$G^{-1}(\xi, \omega + i\delta) = \omega + i\delta - \xi - \Delta^2 \int_{-\infty}^{+\infty} du \frac{\rho(u)}{\omega + i\delta + \xi - u} \tag{10}$$

and the function F is written as

$$F(\xi, u, \omega + i\delta) = G(\xi, \omega + i\delta)[\omega + i\delta + \xi - u]^{-1}. \tag{11}$$

In the clean limit, the scattering effect is taken in account by taking $\delta = 1/2\tau$, where τ is the transport lifetime.

One should stress that the BPT Green function assumes that the vortex lattice is ordered, so that any effect due to loss of long-range order of the vortex lattice, such as a VL state, is completely neglected in our approach.

Carrying out the integration in equation (8), we find

$$I_1 = (4\pi^2\tau/d)[1 + \mu e^{-(2\omega/\varepsilon)^2}]^{-1} \tag{12}$$

where

$$\mu = 2\sqrt{\pi}\tau\Delta^2/\varepsilon = 2\sqrt{\pi}lk(\Delta/\varepsilon)^2 = 2\sqrt{\pi}lk_c(\Delta/\varepsilon)^2[H/H_{c_2}(T)]^{1/2}$$

$k_c = \sqrt{2eH_{c_2}(T)/\hbar}$ and $l = v_F\tau$. v_F is the Fermi velocity. Similarly, integrating equation (9) we find

$$I_2 = (8\pi^{3/2}\Delta^2\tau^2/\varepsilon d)e^{-(2\omega/\varepsilon)^2}[1 + \mu e^{-(2\omega/\varepsilon)^2}]^{-1}. \tag{13}$$

(7) then becomes

$$A_{jj} = [e^2p_F^3/(2\pi)^3m^2v]4\pi^2\tau/d = ne^2\tau/m = \sigma \tag{14}$$

where $n = p_F^2/2\pi d$. On the other hand, considering the expression for $A_{j\Psi}$ given in [2], we can rewrite this quantity for our case as follows:

$$A_{j\Psi} = \frac{2e\Delta p_F}{(2\pi)^3}\int_0^\infty \frac{d\omega}{2T}\operatorname{sech}^2\left(\frac{\omega}{2T}\right) I \tag{15}$$

where

$$I = \int d\xi\, G(\xi, \omega + i\delta)\int d\alpha\, \rho(\alpha)F(\xi, \alpha, \omega - i\delta) \tag{16}$$

with $\rho(\alpha) = \alpha(2/\pi)^{1/2}(4/\varepsilon^2)e^{-(2\alpha/\varepsilon)^2}$. After carrying out the above integration, (15) becomes

$$A_{j\Psi} = (2e\Delta p_F/\pi\sqrt{2})(\tau/\varepsilon d)H(\mu) \tag{17}$$

where

$$H(\mu) = \int_0^\infty \frac{d\omega}{2T}\operatorname{sech}^2\left(\frac{\omega}{2T}\right)\frac{1 - F(2\omega/\varepsilon)}{1 + \mu e^{-(2\omega/\varepsilon)^2}}. \tag{18}$$

The function F is defined as

$$F(x) = 2x^2e^{-x^2}\left[1 + \sum_{n=1}^{\infty}\frac{x^{2n}}{n!(2n+1)}\right]. \tag{19}$$

We can approximate the quantity $D_1(0)\langle[j, \Psi]\rangle(0)$ by the corresponding result obtained by Caroli and Maki in the case of an oscillating transverse field when $\omega, q \to 0$ [12]. This leads to the following expression for σ^{fl}:

$$\sigma^{\text{fl}} = 16\sigma(\Delta/\varepsilon)^2H(\mu). \tag{20}$$

Hence

$$\sigma_s = \sigma^{\text{st}} + \sigma^{\text{fl}} = \sigma[1 + 16(\Delta/\varepsilon)^2H(\mu)]. \tag{21}$$

3. Numerical calculations

In order to evaluate equation (21) numerically, and for future comparison with experimental data on YBCO we take $T_c = 90$ K, $v_F = 10^5$ m s^{-1}, and for $l = v_F\tau$, we consider three different cases:

(i) $\tau = \text{constant} = 0.74\hbar/k_B T_c$;
(ii) $\tau = 0.74\hbar/k_B T$ and (22)
(iii) $\tau = 1.14 \times 10^{-10} e^{-T/12}$. (23)

The expression for τ in (ii) has been deduced from the normal resistance in the a–b plane of monocrystals of YBCO [14]. In (i) τ is taken as constant and equal to its value at T_c given by equation (22). The temperature variation of τ in (iii) has been derived by Bonn *et al* from a microwave surface-resistance study below T_c [9]. We note the semiphenomenological character of cases (ii) and (iii) since a strict BCS model implies that τ is constant.

We derive the quantity $H_{c_2}(T)$ from the BCS upper-critical-field expression worked out by Maki in the case of high-T_c superconductors [15]. The values of $H_{c_2}(T)$ are shown in figure 1. For Δ^2 we consider the following equation derived by Brandt [16] which determines Δ^2 as a function of T and H for pure type-II superconductors in the vicinity of $H_{c_2}(T)$:

$$0 = 2\pi T \sum_{l\geqslant 0}^{\infty} \int_0^{\pi} d\theta \sin\theta \left[\frac{\partial}{\partial\omega} V(x_0)|_{\omega_l} - \frac{\partial}{\partial\omega} V\left(\frac{2\omega}{\varepsilon_0 \sin\theta}\right)\bigg|_{\omega_l} \right] \tag{24}$$

where $dV(x)/dx = W(ix)$. The function $W(z)$ is defined as

$$W(z) = \frac{i}{\pi} \int_{-\infty}^{+\infty} dt\, \frac{e^{-t^2}}{z - t}. \tag{25}$$

$\omega_l = 2\pi T(l + \frac{1}{2})$ is the Matsubara frequency; $\varepsilon_0 = k_c v_F$ and x_0 is determined by

$$2\omega/\varepsilon \sin\theta - x_0 + \sqrt{\pi}(\Delta/\varepsilon \sin\theta)^2 W(ix_0) = 0. \tag{26}$$

Solving equation (26) with $\sin\theta = 1$ yields

$$x_0 = \frac{2\omega}{\varepsilon} + \frac{2(\Delta/\varepsilon)^2}{\alpha + 4(\Delta/\varepsilon)^2} \left[\sqrt{\alpha + 4(\omega/\varepsilon)^2 + 4(\Delta/\varepsilon)^2} - \frac{2\omega}{\varepsilon} \right] \tag{27}$$

where the x_0-dependent number α is comprised between $4/\pi$ and 2 and is given by

$$e^{x_0^2} \int_{x_0}^{+\infty} dt\, e^{-t^2} = \frac{1}{x_0 + \sqrt{x_0^2 + \alpha}}. \tag{28}$$

Using the expression for x_0 above, we compute numerically equation (24) with $\sin\theta = 1$ and thus derive the values of the square of the order parameter Δ^2 for different values of the temperature T and the magnetic field H. The numerical results for Δ^2 as a function of T and H are shown in figures 2 and 3.

Finally, calculating equation (21), we then obtain figures 4–6. In figure 4, $\rho_s(T)/\rho(T_c)$ is reported for the three cases (i), (ii) and (iii) at a magnetic field of 19 T. In figures 5 and 6, $\rho_s/\rho(T_c)$ is shown respectively as a function of T for different magnetic fields, and as a function of H for different temperatures. $\rho_s = \sigma_s^{-1}$ and $\rho^{-1}(T_c) = ne^2\tau(T_c)/m$. We neglect here the Hall conductivity.

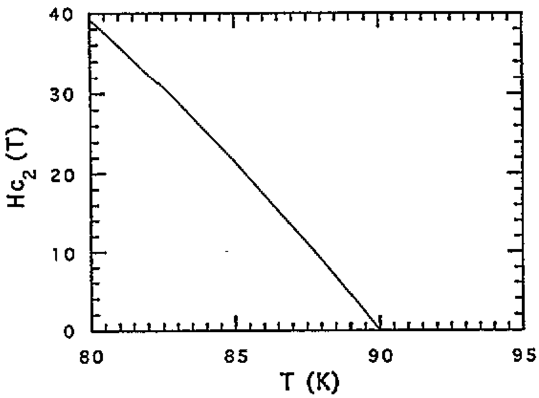

Figure 1. The temperature dependence of H_{c_2} derived from the model used for the present calculations.

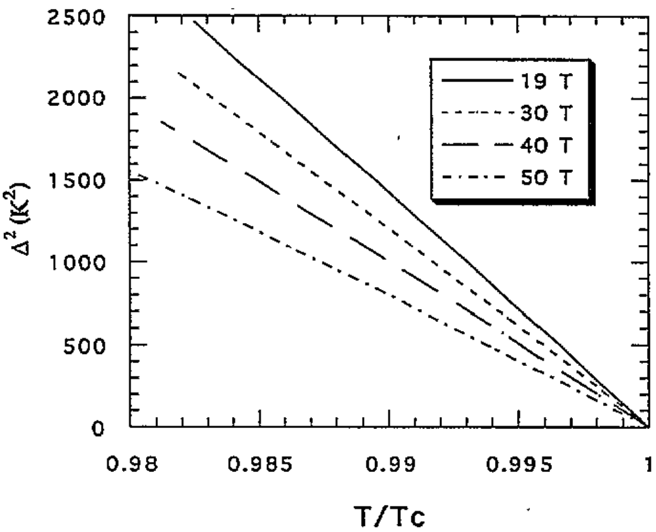

Figure 2. The temperature dependence of the square of the order parameter for different values of the magnetic field.

4. A comparison with experimental data on YBCO

A remarkable feature exhibited by the high-T_c oxides is the anomalous broadening of their resistive transition under a magnetic field. The magnetoresistance $\rho(H \perp I)$ with H perpendicular to I appears initially to increase almost linearly with H, and then saturates after a transition region. Various mechanisms have been suggested to explain the origin of the dissipative process related to these behaviours [17]. The most widely accepted view is that this regime corresponds to a VL phase, which transforms at lower temperature in a pinned vortex glass [3]. In this section, we are interested in the observed experimental behaviours of the $\rho(T, H)$ curves of single crystals or oriented films of YBCO obtained in the field and current geometry relevant to our work.

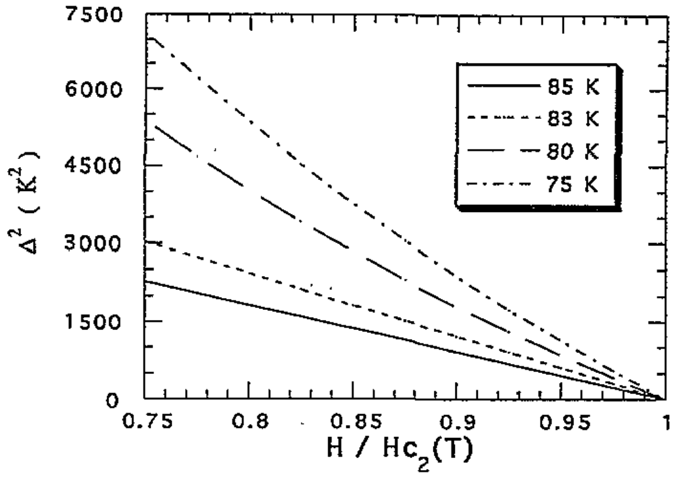

Figure 3. The magnetic field dependence of the square of the order parameter for different values of the temperature.

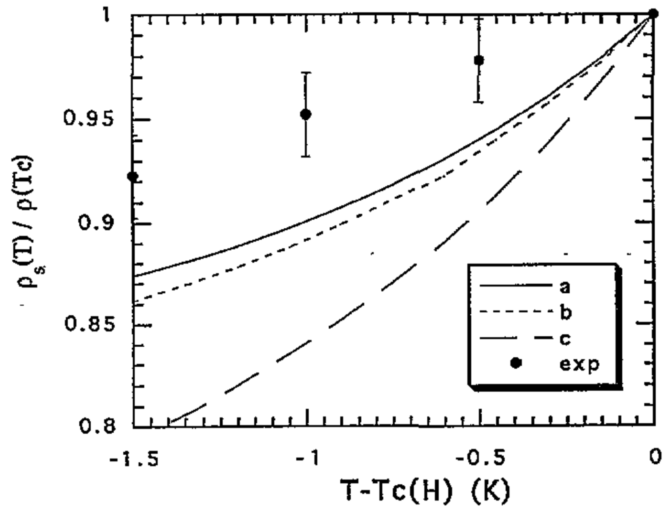

Figure 4. Magnetoresistivity curves near $T_c(H)$ at $H = 19$ T when τ =constant (curve a), $\tau \sim T^{-1}$ (curve b) and $\tau \sim e^{-T/T_0}$ with $T_0 = 12$ K (curve c). The vertical bars represent the experimental results obtained by Oh *et al* [17].

In figure 4, where $\rho_s/\rho(T_c)$ is plotted as a function of $T - T_c(H)$, the vertical bars represent the experimental results of ρ_s obtained by Oh *et al* at 18.9 T in highly oriented films of YBCO when H is perpendicular to the CuO planes [17]. The experimental ρ_s has been normalized by the quantity $\rho(T_c)$ deduced from experiments. Uncertainties in experimental data arise mainly from the determination of T_c. The theoretical curves a, b and c in figure 4 exhibit a clear broadening of the resistive transition. There is only a slight difference between curves a and b, while curve c shows a remarkable difference with regard to curves a and b. We note also that quantiative agreement is not improved

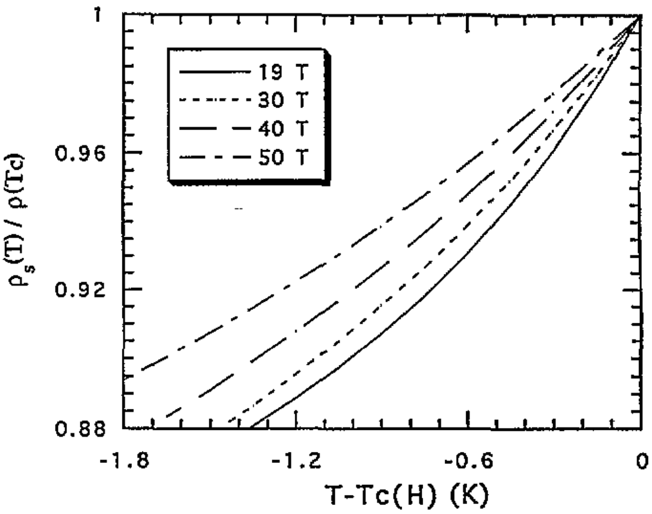

Figure 5. Resistivity curves near $T_c(H)$ for different values of the magnetic field when τ is constant

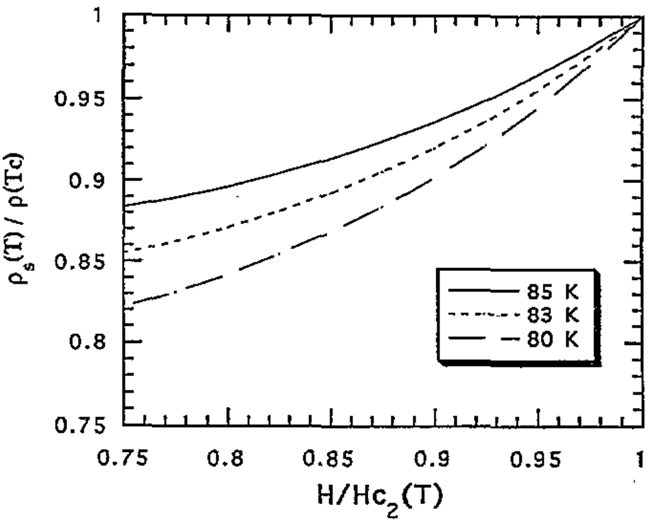

Figure 6. Magnetoresistivity curves near $H_{c_2}(T)$ for different values of the temperature when τ is constant.

when temperature-dependent relaxation rates are considered. In figure 5, $\rho_s(T)/\rho(T_c)$ is reported for different values of H, when τ is constant (case (i)). The curves show a resistive transition broadening, as well as a clear decrease of the slope with increasing field. Figure 6 depicts the variations of ρ_s versus H at various temperatures when τ is constant (case (i)). We note the small deviations from linear behaviour shown by the curves $\rho_s(H)$ at different temperatures. For the various temperatures considered, when H varies from $H_{c_2}(T)$ to $0.75H_{c_2}(T)$, $\rho_s(H)/\rho(T_c)$ presents only a variation of about 2% T^{-1}, indicating a saturation-like behaviour of the $\rho(H)$ curves at different temperatures in the vicinity of

$H_{c_2}(T)$. When $\tau \sim T^{-1}$ (case (ii)) or $\tau \sim e^{-T/12}$ (case (iii)) we find qualitatively the same behaviours for the curves $\rho_s(H)/\rho(T_c)$ at different temperatures and for the curves $\rho_s(T)/\rho(T_c)$ at different magnetic fields.

5. Discussion and conclusions

Our calculation, which takes into account the dynamical fluctuations of the order parameter in the otherwise rigid VLL, extends to layered compounds and finite T the 3D and 0 K calculations of Maki and Houghton [2]. It is based on the clean limit of the BCS model and the high-field approach of Brandt *et al* [13], which describes the incipient Abrikosov lattice in the vicinity of $H_{c_2}(T)$. When compared with experiments, our results account for the broadening of the resistive transition and the decrease of the slope $d\rho_s(T)/dT$ with increasing field, as well as the saturation-like behaviour of the magnetoresistivity curves near $H_{c_2}(T)$. However, the 'most dissipative' result, which corresponds to a constant τ, yields a too steep decrease of the resistivity with T when compared to experiments. The observed flux-flow resistivity is larger than the theoretical result by at least 5% (about 15% for the case of a sharp increase in τ as T decreases below T_c—choice (iii)).

In high-T_c cuprates, a temperature-independent τ is not likely. The normal-state resistivity is dominated by inelastic processes, which are very likely to be strongly affected by the superconducting transition. Thus our result for constant τ is quite probably a minimum estimate of the conductivity gain within the VLL, due to the order-parameter dynamic fluctuations. The difference between the experimental result and our theoretical one may be ascribed to the vortex lines' fluctuative thermal motion. It is striking that within the temperature interval we consider, this latter term is only about 5% to 15% of the observed dissipation. Our calculation does not reliably allow us to explore the intermediate temperature ranges. However it suggests that it is dangerous to interpret the observed flux-flow conductivity in high-T_c cuprates as due essentially to VL effects. If this is so, the question arises of the mechanism behind the transition to a zero-resistivity state at finite temperature in the mixed phase, ascribed within phenomenological theories to the formation of vortex glass [3]. Within a microscopic approach, such a transition must be associated with a qualitative change of the low-frequency excitation spectrum, possibly because of collective effects connected to pinning. Whether this is compatible with the vortex-glass concept is an open question.

An obvious shortcoming of our microscopic analysis stems from the fact that results obtained with 'experimental' relaxation rates do not lead to a better quantitative agreement with experimental data. Note, however, that the 'experimental' relaxation rate is obtained in a zero-field state [9], so it may have little to do with the relaxation rate in the mixed phase. Again, this points to the necessity of a truly microscopic theory of high-T_c superconductivity. In particular, it would be useful to take into account the d-wave symmetry of the order parameter. It is likely that dissipation is increased with d-wave symmetry as compared to s wave, so that the level of dissipation within the framework of the theory described in this paper might be closer to experiment; the part of the flux-flow resistivity due to VL effects might thus be even smaller than estimated above.

In conclusion, our microscopic theory, in spite of its several important drawbacks, suggests that the appropriate approach to discussing flux-flow conductivity close to $H_{c_2}(T)$ and $T_c(H)$ is to consider the enhancement of the normal-state conductivity due to order-parameter fluctuations, rather than the dissipation due to the thermal fluctuations of a vortex liquid. A clear picture of the physics involved requires, however, a fully microscopic theory of high-T_c superconductivity.

Acknowledgments

We have immensely benefited from a very thorough (anonymous) referee's critical work dealing with a previous version, which has allowed us to improve this work significantly. We would like to thank this unknown contributor to our work. We have had useful discussions with Ian Campbell and Sadoc Senoussi. We thank Gianni Blatter for a useful comment. This work is supported in part by CEE contract Human Capital and Mobility No CHRX-CT93-0332.

Appendix

We consider a superconductor on which is applied a transverse vector potential $A(A_x, 0, 0)$. The perturbing Hamiltonian associated with A gives rise to fluctuations of the order parameter, which leads to change in the average value of the current operator

$$j_x = -\frac{\mathrm{i}}{2m}\sum_{\sigma}\nabla_x\psi_\sigma^+(r)\psi_\sigma(r) - \psi_\sigma^+(r)\nabla_x\psi_\sigma(r). \tag{A1}$$

$\psi_\sigma(r)$ and $\psi_\sigma^+(r)$ are the electron field operators. In linear-response theory the resulting current density is given by $j_x(q,\omega) = Q_{xx}(q,\omega)A_x(q,\omega)$ where the quantity Q_{xx} can be written as follows for $vq \to 0$:

$$Q_{xx}(-\mathrm{i}\omega) = \langle[j_x, j_x]\rangle(-\mathrm{i}\omega) + 2(\langle[j_x, \Psi]\rangle D_1\langle[\Psi, j_x]\rangle)(-\mathrm{i}\omega) \tag{A2}$$

with $\Psi(r,t) = \psi_\uparrow(r,t)\psi_\downarrow(r,t)$ and $D_1(-\mathrm{i}\omega) = |g|/[1 - |g|\langle[\Psi, \Psi^+]\rangle(-\mathrm{i}\omega)]$. $\langle[\,]\rangle$ denotes the average value of the retarded product taken on the Gibbs ensemble of the unperturbed state of the system. The flux-flow conductivity is then given by

$$\sigma_{xx} = \lim_{\omega\to 0}(Q_{xx}(-\mathrm{i}\omega)/-\mathrm{i}\omega). \tag{A3}$$

In a DC electric field, $\langle[j_x, j_x]\rangle(-\mathrm{i}\omega)$, $\langle[j_x, \Psi]\rangle$ can be expanded in powers of ω as shown by equations (4) and (5) and for D_1 we have

$$D_1(-\mathrm{i}\omega) = D_1(0) - \mathrm{i}\omega D_1^{(1)}. \tag{A4}$$

Neglecting $D_1^{(1)}$, which is smaller than A_{jj} and $A_{j\psi}$ by a factor ξ/l and reporting the results of equations (4), (5) and (A4) in (A3) we recover equations (1), (2) and (3).

References

[1] Houghton A and Maki K 1991 *Phys. Rev.* B 4 843
[2] Maki K and Houghton A 1971 *Phys. Rev.* 4 847
[3] See e.g.
Blatter G, Feigel'man M V, Geshkenbein V B, Larkin A I and Vinokur V M 1994 *Rev. Mod. Phys.* at press
[4] Kamaras K *et al* 1990 *Phys. Rev. Lett.* 64 84
[5] Biéri J B and Maki K 1990 *Phys. Rev.* B 42 4854
[6] Biéri J B, Maki K and Thompson R S 1991 *Phys. Rev.* B 44 4709
[7] Reiter G, Horsch P and Psaltakis G 1991 *Dynamics of Magnetic Fluctuations in High T Superconductors (NATO ASI Series)* (New York: Plenum)

[8] See e.g.
Nagaosa N and Lee P A 1990 *Phys. Rev. Lett.* **64** 2450
Grilli M and Kotliar G 1990 *Phys. Rev. Lett.* **64** 1170
Nagaosa N and Lee P A 1992 *Phys. Rev.* B **45** 10719
[9] Klein O, Holczer K and Grüner G 1992 *Phys. Rev. Lett.* **68** 2407
Bonn D A *et al* 1993 *Phys. Rev.* B **47** 11314
[10] Tsuei C C, Kirtley J R, Chi C C, See Yu-Jahnes L, Gupta A, Shaw T, Sun J Z and Ketchen M B 1994 *Preprint*
[11] Caroli C and Maki K 1967 *Phys. Rev.* **159** 306
[12] Caroli C and Maki K 1967 *Phys. Rev.* **159** 316
[13] Brandt U, Pesch W and Tewordt L 1967 *Z. Phys.* **201** 209
[14] Batlogg B 1990 *High Temperature Superconductivity* ed K S Bedell *et al* (Redwoood City, CA: Addison Wesley) p 37
[15] Maki K 1991 unpublished
[16] Brandt U 1969 *Phys. Lett.* **29A** 568
[17] Oh B, Char K, Kent A D, Naito M, Beasley M R, Geballe T H, Hammond R H and Kapitulnik A 1988 *Phys. Rev.* B **37** 7861
Li J N, Kadowaki K, Menken M J V, Menovsky A A and Franse J J M 1989 *Physica* C **161** 313
Palstra T M, Batlogg B, Schneemeyer L F and Waszczak J V 1990 *Phys. Rev. Lett.* **64** 3090
Kwok W K, Welp U, Crabtree G W, Vandervoort K G and Liu J Z 1990 *Physica* B **163** 242
Worthington T K, Holtzberg F H and Feild C A 1990 *Cryogenics* **30** 417
Martinez J C, Laborde O, Préjean J J, Chappert C, Renard J P, Karpinski J and Kaldis E 1991 *Europhys. Lett.* **14** 693